1 Postulates of Special Relativity

Einstein’s theory of Special Relativity was first unveiled in 1905, and was based upon two very basic but very profound postulates:

- The laws of physics are the same in all inertial frames of reference
- The speed of light $c$ is invariant, i.e. it has the same value in all inertial frames of reference

Central to both of these postulates is the concept of an inertial frame, which can be defined as a frame in which an observer experiences no acceleration, i.e. is at rest or travelling at a constant velocity.

2 Lorentz Transformations

If an inertial frame of reference $S$ is at rest and a second frame $S'$ is traveling away from $S$ at a speed $\beta = v/c$ in the x-direction, then the coordinates $(x, y, z, ct)$ in $S$ and $(x', y', z', ct')$ in $S'$ are related by the Lorentz Transformations:

$$x' = \gamma(x - \beta ct)$$

$$y' = y$$

$$z' = z$$

$$ct' = \gamma(ct - \beta x)$$  \(1\)
where the relativistic factor $\gamma$ is given by:
\[
\gamma = \frac{1}{\sqrt{1 - \beta^2}} = \frac{1}{\sqrt{1 - v^2/c^2}}
\]  
(2)

This gives rise to the inverse Lorentz Transformations:
\[
x = \gamma(x' + \beta ct')
\]
\[
ct = \gamma(ct' + \beta x')
\]
(3)

In all frames of reference one can define an invariant quantity, the Lorentz invariant:
\[
\Delta s^2 = -\Delta \tau^2 = \Delta x^2 + \Delta y^2 + \Delta z^2 - c^2 \Delta t^2
\]
(4)

If we consider a particle in frame $S$ moving at velocity $\vec{u} = u_x \hat{x} + u_y \hat{y}$, then its relativistic velocity components in $S'$ are given by:
\[
u_x' = \frac{u_x - v}{1 - vu_x/c^2}
\]  
(5)
\[
u_y' = \frac{u_y}{\gamma(1 - vu_x/c^2)}
\]  
(6)

3 Time Dilation and Length Contraction

If an object in its rest frame is measured to have length $L_0$, then an observer moving toward the object with velocity $v$ and relativistic factor $\gamma$ measures the objects length as $L$ given by:
\[
L = L_0/\gamma = L_0 \sqrt{1 - v^2/c^2}
\]
(7)

If an observer at rest measures a time $t_0$ to have passed, then an observer in another inertial frame will measure time $t$ given by:
\[
t = \gamma t_0 = t_0/\sqrt{1 - v^2/c^2}
\]
(8)

4 Dynamics

If an object at rest is measured to have rest mass $m_0$, then the mass of the object when traveling at velocity $v$ is given by:
\[
m = \gamma m_0 = \frac{m_0}{\sqrt{1 - v^2/c^2}}
\]
(9)
and has a momentum $p$ given by:

$$p = \gamma m_0 v = \frac{m_0 v}{\sqrt{1 - v^2/c^2}}$$  \hspace{1cm} (10)$$

and relativistic energy:

$$E = mc^2 = \gamma m_0 c^2$$  \hspace{1cm} (11)$$

These quantities can be related to one another by the mass invariant:

$$(m_0 c^2)^2 = E^2 - (pc)^2$$  \hspace{1cm} (12)$$

If the mass is subject to a force $F_x$ in the x-direction resulting in an acceleration $a_x$ in the frame $S$, then these quantities are related by:

$$F_x = \gamma^3 m_0 a_x$$  \hspace{1cm} (13)$$