GRE Special Relativity Review

Socity of Physics Students

October 17, 2004

1 Postulates of Special Relativity

Einstein's theory of Special Relativity was first unveiled in 1905, and was based upon two very basic but very profound postulates:

- The **laws of physics are the same** in all inertial frames of reference
- The **speed of light** c **is invariant**, i.e. it has the same value in all inertial frames of reference

Central to both of these postulates in the concept of a **inertial frame**, which can be defined as a frame in which an observer experiences no acceleration, i.e. is at rest or travelling at a constant velocity.

2 Lorentz Transformations

If an inertial frame of reference S is at rest and a second frame S' is traveling away from S at a speed $\beta = v/c$ in the x-direction, then the coordinates (x, y, z, ct) in S and (x', y', z', ct') in S' are related by the Lorentz Transformations:

$$\begin{aligned} x' &= \gamma(x - \beta ct) \\ y' &= y \\ z' &= z \\ ct' &= \gamma(ct - \beta x) \end{aligned} \tag{1}$$

where the relativistic factor γ is given by:

$$\gamma = \frac{1}{\sqrt{1-\beta^2}} = \frac{1}{\sqrt{1-v^2/c^2}} \tag{2}$$

This gives rise to the inverse Lorentz Transformations:

$$\begin{aligned} x &= \gamma(x' + \beta ct') \\ ct &= \gamma(ct' + \beta x') \end{aligned} \tag{3}$$

In all frames of reference one can define an invariant quantity, the Lorentz invariant:

$$\Delta s^2 = -\Delta\tau^2 = \Delta x^2 + \Delta y^2 + \Delta z^2 - c^2\Delta t^2 \tag{4}$$

If we consider a particle in frame S moving at velocity $\vec{u} = u_x\hat{x} + u_y\hat{y}$, then its relativistic velocity components in S' are given by:

$$u'_x = \frac{u_x - v}{1 - vu_x/c^2} \tag{5}$$

$$u'_y = \frac{u_y}{\gamma(1 - vu_x/c^2)} \tag{6}$$

3 Time Dilation and Length Contraction

If an object in its rest frame is measured to have length L_0, then an observer moving toward the object with velocity v and relativistic factor γ measures the objects length as L given by:

$$L = L_0/\gamma = L_0\sqrt{1-v^2/c^2} \tag{7}$$

If an observer at rest measures a time t_0 to have passed, then an observer in another inertial frame will measure time t given by:

$$t = \gamma t_0 = t_0/\sqrt{1-v^2/c^2} \tag{8}$$

4 Dynamics

If an object at rest is measured to have rest mass m_0, then the mass of the object when traveling at velocity v is given by:

$$m = \gamma m_0 = \frac{m_0}{\sqrt{1-v^2/c^2}} \tag{9}$$

and has a momentum p given by:

$$p = \gamma m_0 v = \frac{m_0 v}{\sqrt{1 - v^2/c^2}} \tag{10}$$

and relativistic energy:

$$E = mc^2 = \gamma m_0 c^2 \tag{11}$$

These quantities can be related to one another by the mass invariant:

$$(m_0 c^2)^2 = E^2 - (pc)^2 \tag{12}$$

If the mass is subject to a force F_x in the x-direction resulting in an acceleration a_x *in the frame* S, then these quantities are related by:

$$F_x = \gamma^3 m_0 a_x \tag{13}$$