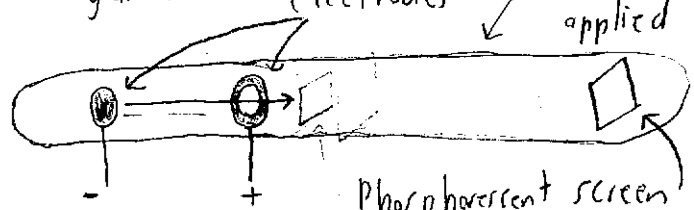
T. Mooring

Notes on Atomic Physics ①

8.04

Atoms - several Å big
 Å = 10^{-10} m
Planck const. - h - energy × time
 ↳ * New + important
QM important
 when system "small"
 momentum × size ≤ h
CGS units
 Erg - g cm^2 s^{-2}
 Dyne - g cm s^{-2}
 Charge - esu
 ↳ * eliminates constants in electrostatic force law
Electron Volt (eV)
 ↳ 1.6×10^{-12} erg (approx.)
Atomic mass unit (amu)
 ≈ 1 H atom
 = 1/12 ^{12}C atom
Mol - amount of substance for which total mass in g equals molecule's mass + in amu
Faraday - Unit of charge
 ↳ 96485 C
Avogadro's # - # molecules in mol of substance -

Electron Discovery - J.J Thomson, 1897
 "Cathode rays" in low pressure gas tube



* $\vec{E}$ + $\vec{B}$ fields are imposed on the tube (not shown) ⊥ to each other + the electron path
* We thus measure the electron charge-to-mass ratio (first velocity, then e/m_e)

$e = 4.80324 \times 10^{-10}$ esu $= 1.6022 \times 10^{-19}$ C
$m_e = 9.109 \times 10^{-28}$ g

Thomson atom - Ball of + charge w/ e^- embedded
 "Ball size" (atomic radius) estimatable from molar mass, density, + Avogadro's #
 * Does not explain spectra well

Electron Spin - Stern-Gerlach Exp.
 Electrons have spin → feel force in inhomogenous $\vec{B}$ field
 ↳ $U = \vec{\mu} \cdot \vec{B} \rightarrow \vec{F} \propto \nabla(\vec{\mu} \cdot \vec{B})$
 * e^- deflect to only 2 places on target screen
In double slit exp., e^- give wave interference pattern

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Notes on Atomic Physics ②

Spectroscopy of H

Balmer series

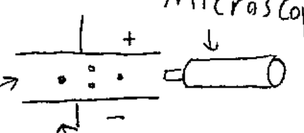
$$\frac{1}{\lambda_n} = R_H\left(\frac{1}{4} - \frac{1}{n^2}\right)$$

$n = 3, 4, 5, \ldots$

R_H - Rydberg const.

↳ $109700\ cm^{-1}$

Millikan Oil Drop exp.



Use

1) $\vec{E}$ field off, use drop fall rates to find radii + masses
2) Turn on field, use upward terminal velocities + masses to find charges
3) Charge turns out to be quantized

* Atoms of widely varying mass have roughly same volume

More Spectroscopy

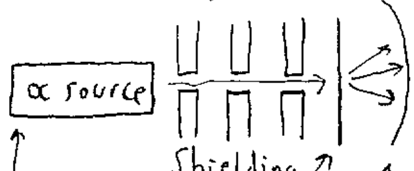
$$\frac{1}{\lambda} = R\left(\frac{1}{m^2} - \frac{1}{n^2}\right)$$

↳ const. for each atom

$n > m$

Rutherford Scattering

Discovery of nucleus



~100 atom thick Au foil

Detector measures scattering angle

* The Thomson model predicts far too few large-angle scatters
* Thus + charge resides in 10^{-13} cm nucleus, surrounded by 10^{-8} cm e^- cloud

Blackbody Radiation

↳ perfect absorber emits thermally

Radiated intensity $= \sigma T^4$

$\sigma = 5.67 \times 10^{-5}$ erg $K^{-4} cm^{-2} s^{-1}$

↳ also $\int_0^\infty R_\nu(\nu, T)\,d\nu$
↳ intensity / unit freq

* ν @ $\max(R_\nu, T) \propto T$

What is $R_\nu(\nu, T)$?

* Consider modes in cubical box

Assume modes can have any energy E

Then use Boltzmann dist to find $\langle E \rangle = \int_0^\infty E p(E)\,dE$

$= k_B T$

Count modes in the cubical box

* Unfortunately, as $\nu \to \infty$, $R_\nu(\nu, T) \to \infty$
↳ Rayleigh - Jeans spect.

Let's suppose that energy in mode can increase only in steps of

$\Delta E = h\nu$
↳ some constant

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Notes on Atomic Physics ③

More Blackbody Radiation

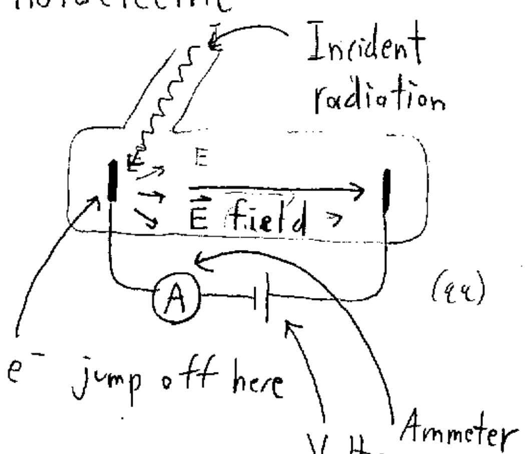
Then $p(E_n) = \frac{\exp[-E_n/k_B T]}{N}$ (normalization → N)

$\langle E \rangle = \sum_{n=0}^{\infty} E_n p(E_n) = \frac{h\nu}{\exp[h\nu/k_B T]-1}$

* $h = 6.626 \times 10^{-27}$ erg·s fits experimental data

** Very important

Photoelectric Effect



* We vary ν of incident light + applied $\vec{E}$ field

* Measure current, lets us find # + max. kinetic energy of ejected e^-

Results

1) K_{max} independent of light intensity
2) K_{max} increase w/ ν
3) No e^- for $\nu <$ some ν_0 – metal dependent

* Not classically expected

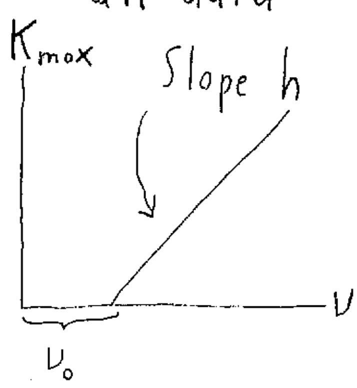
Einstein (1905)

Light quantized in photons w/

$E = h\nu$

$h\nu_0$ is required to eject an e^-

* This explains all data

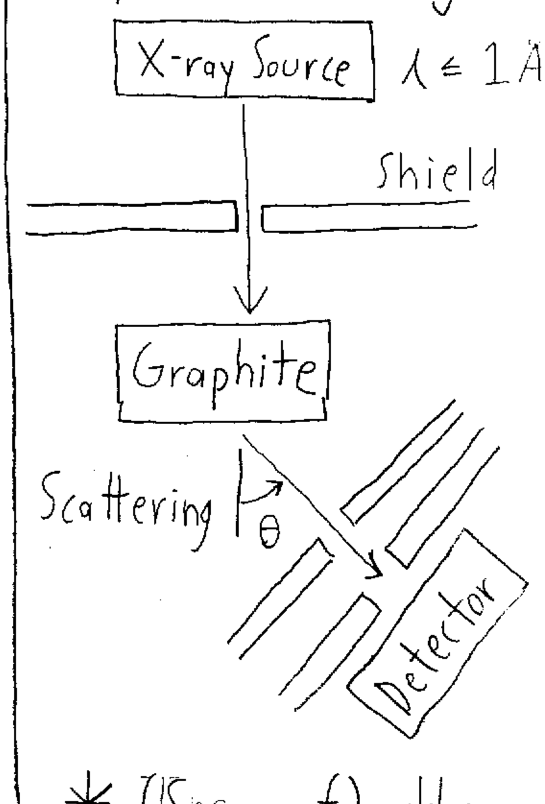


* $h = 6.6262 \times 10^{-27}$ erg·s ($\times 10^{-34}$ J·s)

Rayleigh Scattering

Bound atomic e^- vibrate b/c of incident EM radiation, radiate @ same ν as result

Compton Scattering



* (Some of) the scattered X-rays are wavelength shifted (cf. Rayleigh)

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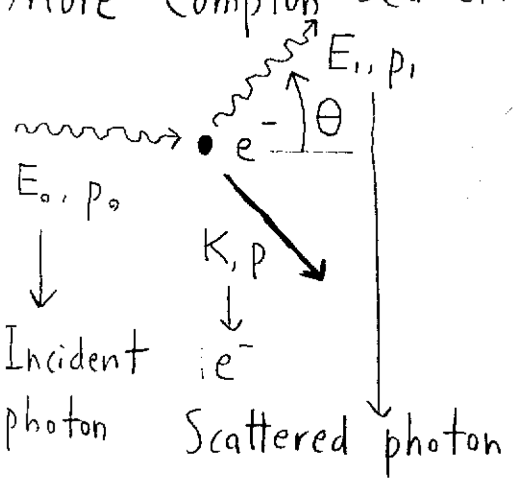
Notes on Atomic Physics ④

More Compton Scattering



$E^2 = m^2c^4 + p^2c^2$

$E_{photon} = pc \quad p = h/\lambda$

* Compton shift

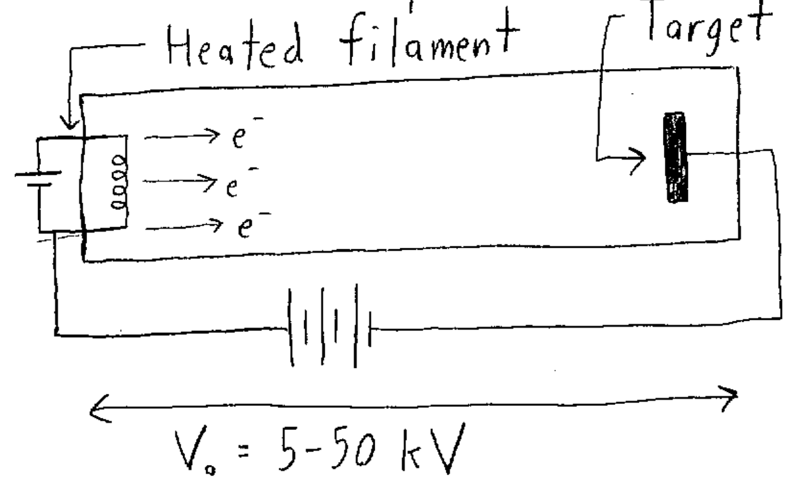
$\Delta\lambda = \lambda_1 - \lambda_0 = \frac{h}{m_e c}(1 - \cos\theta)$

This fits data

However Rayleigh prediction not totally wrong — for inner shell, effective $m > m_e$, $\Delta\lambda \to 0$

Bremsstrahlung ("stopping radiation")

Short-λ X-rays



* e^- jump off filament, release X-rays when hitting metal target
* B/c each e^- has only energy eV_0, there is lower bound to X-ray λ's → cf. classical theory
* $h\nu_{max} = eV_0$

De Broglie Relation

Light like particle → perhaps matter like wave

**** $\lambda = h/p$ for everything

Notes on Atomic Physics ⑤

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Rutherford - Bohr (H) Atom

Posit:

1) e^- can be in certain orbits obeying Newtonian mech. but w/o radiating
2) \\(" stationary states")
2) When e^- jumps between states, photon w/ E = energy difference between states is emitted

* Easily show that

$$E_{photon} = \frac{e^2}{2}\left[\frac{1}{r_f} - \frac{1}{r_i}\right]$$

Initial + final orbits

* Balmer series → $r_i \propto n^2$

$r_n = a_0 n^2$ ← (a_0 → Bohr rad.)

* Can show

$$a_0 = \frac{e^2}{2hcR_H}$$

* Require classical + quantum agreement @ large n →

$$\nu^2_{photon} \approx \frac{e^4}{h^2 a_0^2 n^4}$$

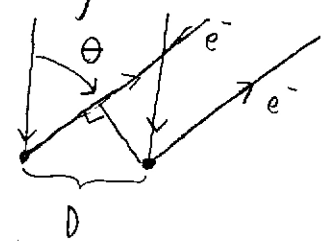
* Posit $\nu_{photon} = \nu_{orbiting\ e^-}$ → get

$$a_0 = \frac{h^2}{4\pi^2 m_e e^2}$$

* We have (reasonably correctly) "derived" R_H, the H size, + ionization energy!!!! * * * *

Davisson - Germer Exp.

Scattering of e^- off top layer of Ni crystal confirmed De Broglie relation ↓



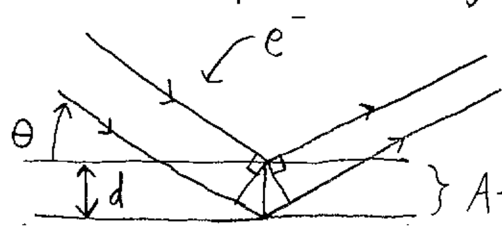
Need $\lambda = D\sin\theta$ for scattering max.

Also for photons: $E = \hbar\omega$

For everything: $p = \hbar k$

$$\hbar = \frac{h}{2\pi}$$

More Davisson - Germer (multi - layer scattering)



Maxima ↓

$$\sin\theta_n = \frac{n\lambda}{2d}$$

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Notes on Atomic Physics ⑥

More Bohr Atom

With additional postulate that ang. mom. is quantized in units of $\hbar$ (so $L = n\hbar$) can > easily derive

* $a_0 = \hbar^2 / e^2 m_e$
* Then show nth orbit energy $E_n = -\frac{Z^2 e^4 m_e}{2\hbar^2}\frac{1}{n^2}$

$$= -\frac{\alpha^2 mc^2}{2}\left(\frac{Z}{n}\right)^2$$

*** Fine struct. const.

$$\alpha \equiv \frac{e^2}{\hbar c} \approx \frac{1}{137}$$

Lyman series: to $n = 1$
Balmer " " $n = 2$
Paschen " " $n = 3$

Wilson - Sommerfeld Quantiz.

$$\oint p(q)\,dq = nh$$

* system periodic in q, has conjugate mom. p

* Double - Slit Exp.

When passing light (or particles) through 2 appropriate slits, observed intensity pattern is NOT Σ of intensity patterns for each slit

*****
Heisenberg Uncertainty!

$\Delta x \Delta p_x \geq \hbar/2$

$\Delta E \Delta t \geq \hbar/2$

$\Delta x \Delta k \geq 1/2$

- Can use to estimate ground state energies by constraining p or x