Physics GRE Review Fall 2004
General Relativity Problems

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Special Relativity Problem Set

The standard MIT relativity curriculum, either from 8.033 or 8.20, far exceeds the level of detail tested on the GRE. This problem set is intended to refresh the essential concepts you can expect to encounter on the Physics GRE.

1. **Relativistic Coordinate Transformation**
An event occurs in a moving frame S' at coordinates $(x', y', z', ct') = (3m, 1m, 0, 2m)$. S' moves at velocity $v = 0.8c$ in the x-direction with respect to a rest frame S. Find the coordinates of the event in S.
(A)$(\mathbf{7.66, 1, 0, 7.33})$
Here one need simply apply the Lorentz Transformations:

$$\begin{aligned} x = \gamma(x' + \beta ct') = \frac{5}{3}(3 + 0.8 * 2) = 7.66 \\ y = y' = 1 \\ z = z' = 0 \\ ct = \gamma(ct' + \beta x') = \frac{5}{3}(2 + 0.8 * 3) = 7.33 \\ \Rightarrow (x, y, z, ct) = (7.66, 1, 0, 7.33) \end{aligned}$$

2. **Length Contraction**
If a moving observer holds a rod of length $L_0 = 1m$ at an angle of $\theta = 45^o$ from the x-axis, and is travelling at velocity $v = \sqrt{1/2}c$, find the length as measured by a stationary observer.
(B)$\sqrt{\mathbf{5/8}}\mathbf{m}$
To do this problem one must recall that length conctraction only occurs *along the direction of motion.* This means that one must break up the bar's length into parallel and perpendicular components. The perpendicular component is unchanged:

$$L_{perp} = L_{0,perp} = L_0 \sin\theta = 1m/\sqrt{2} \tag{1}$$

while the parallel component is contracted:

$$L_{||} = L_{0,||}/\gamma = L_0 \cos\theta/\gamma = 1m/2\sqrt{2} \tag{2}$$

Thus the total length is determined by the two:

$$L = \sqrt{L_{perp}^2 + L_{||}^2} = \sqrt{5/8}m \tag{3}$$

3. **Time Dilation**
If a moving observer sees a time of $t_0 = 3s$ elapse on his own watch and a stationary observer measures $t = 5s$ for the same time interval, find the relative velocity of the two observers.

(**E**)**0.8c**
Here one must use the formula for time dilation:

$$\begin{aligned} t = \gamma t_0 &\Rightarrow \gamma = t/t_0 \\ \Rightarrow \frac{1}{\sqrt{1-(v/c)^2}} &= 5/3 \\ &\Rightarrow v = 0.8c \end{aligned}$$

4. **Relativistic Velocity Addition**
A particle in rest frame S moves at velocity $\vec{u} = (0.8c, 60^o)$. Find the particles velocity u' in frame S' which moves at a velocity $\vec{v} = v\hat{x}$ with respect to S.

(**D**)$(\mathbf{0.775c}, \mathbf{110^o})$
It is first useful to determine the velocity components in frame S:

$$\begin{aligned} u_x &= 0.8c\cos(60^o) = 0.4c \\ u_y &= 0.8c\sin(60^o) = 0.4\sqrt{3}c \end{aligned}$$

One can then find the velocity components in S' by plugging these values into the relativistic velocity addition equations:

$$\begin{aligned} u'_x &= \frac{u_x - v}{1 - \frac{u_x}{c}\frac{v}{c}} = \frac{0.4 - 0.6}{1 - 0.4 * 0.6} = -0.263c \\ u'_y &= \frac{u_y}{\gamma(1 - \frac{u_x}{c}\frac{v}{c})} = \frac{0.4\sqrt{3}}{1.25 * (1 - 0.4 * 0.6)} = 0.729c \end{aligned}$$

from which one can find the new velocity and trajectory:

$$(v', \theta') = (0.775c, 110^o) \tag{4}$$

5. **Relativistic Collision**
A particle with rest mass m_0 travels at a velocity $v \approx c$ with relativistic factor $\gamma_v = 100$ and collides with a second particle of mass m_0. If the two particles stick together upon collision, find the factor γ_u for the velocity u with which the particles travel after the collision.

(C)$\gamma_{\mathbf{u}} = \mathbf{50}$
The key to this problem is to use conservation of relativistic momemtum. The momentum of the system before the collision is given by:

$$p_i = \gamma_v m_0 v \approx \gamma_v m_0 c \tag{5}$$

where the approximation $v \approx c$ is used because $\gamma_v = 100$ is so large. The momentum after the collision is:

$$p_f = \gamma_u * 2m_0 * u \tag{6}$$

So far, we are not sure if we can approximate $u \approx c$, so we will try to solve for u and then find γ_u. To do this, equate the initial and final momentum (and use the notation $\beta = u/c$):

$$\begin{aligned} \gamma_v m_0 c &= \frac{2m_0 u}{\sqrt{1-(u/c)^2}} \\ \gamma_v &= \frac{2\beta}{\sqrt{1-\beta^2}} \\ \frac{\gamma_v^2}{4} &= \frac{\beta^2}{1-\beta^2} \\ \beta^2 = 1 - 1/(\gamma_v/2)^2 &\approx 1 \end{aligned}$$

Thus we have shown $u \approx c$, so we can now go back to our momentum conservation equation to find γ_u:

$$\begin{aligned} \gamma_v m_0 c = \frac{2m_0 u}{\sqrt{1-(u/c)^2}} &\approx 2\gamma_u m_0 c \\ \Rightarrow \gamma_u &\approx \gamma_v/2 = 50 \end{aligned}$$