# Notes on Waves

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## 1 Superposition

For $x_1 = A\cos\omega_1 t$ and $x_2 = A\cos\omega_2 t$, the superposition $x = x_1 + x_2$ can be written as

$$x = 2A\cos\frac{\omega_1 - \omega_2}{2}t\cos\frac{\omega_1 + \omega_2}{2}t. \tag{1}$$

"Beating" occurs when $|\omega_1 - \omega_2| << \omega_1 + \omega_2$. The envelope of the beats is $x = \pm 2A\cos\frac{\omega_1 - \omega_2}{2}t$.

## 2 Interference and Diffraction

### 2.1 Double-slit interference

See Figure 1 for a picture of the geometry involved. Interference maxima occur at $\sin\theta_n = \frac{n\lambda}{d}$.

### 2.2 Single-slit diffraction

Looks like $A = A_0\frac{\sin\phi/2}{\phi/2}$, where $\frac{\phi}{2} = \frac{\pi b\sin\theta}{\lambda}$, and $b$ is the size of the slit. See Figure 2.

## 3 Boundary Effects

For a pulse traveling along a line, if it hits a fixed end ($Z = \infty$), it will reflect back in the opposite direction. If it hits a free end ($Z = 0$), it will reflect back in the same direction. If a wave travels from a material with impedance $Z_1$ to one with impedance $Z_2$, the reflection coefficient is

$$\Gamma = \frac{Z_1 - Z_2}{Z_1 + Z_2}. \tag{2}$$

## 4 Doppler Effect

Relativistic limit: $f_0 = \sqrt{\frac{1-v/c}{1+v/c}}f_s$, where c is the speed of light, v is the speed of the object away from the source, $f_0$ is the frequency observed, and $f_s$ is the frequency emitted. For the classical limit: $f_0 = (1 - v/c)f_s$. Remember that things moving away from you redshift.

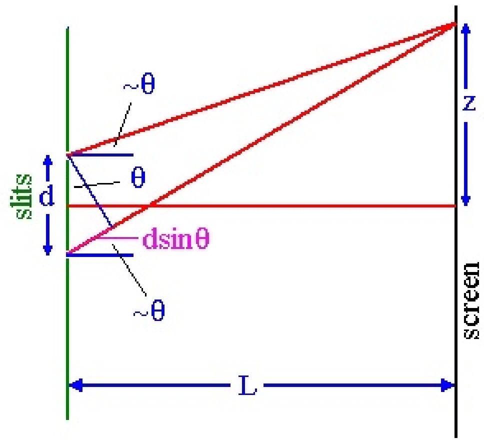


Figure 1: Use these relations to find the maxima and minima of the double-slit interference pattern. (Source: phys.utk.edu)

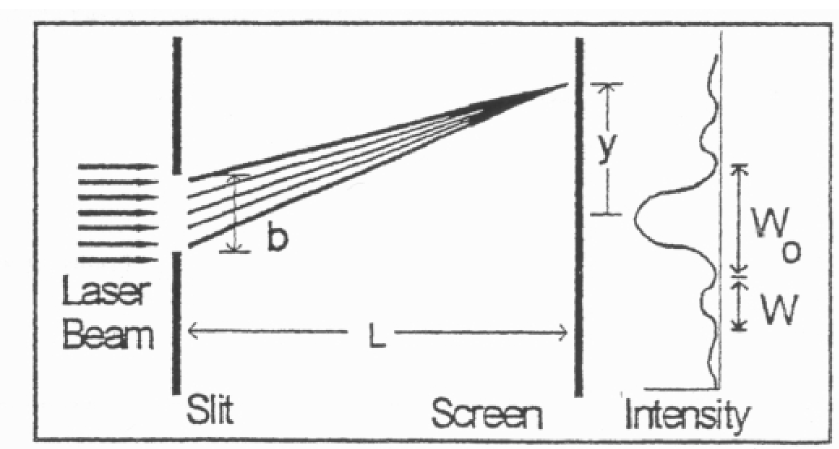


Figure 2: Single slit diffraction (Source: phoenix.phys.clemson.edu)

Properties of wave systems

| System | $v$ | $Z$ |
|---|---|---|
| string | $\sqrt{T/\mu}$ | $\sqrt{T\mu}$ |
| torsional wave | $\sqrt{K/I}$ | $\sqrt{KI}$ |
| transmission line | $1/\sqrt{LC}$ | $\sqrt{L/C}$ |

# 5 Systems

See Table 5.

# 6 Telescopes

The size of a telescope required to achieve a certain angular resolution (for diffraction- limited systems) is

$$\theta = 1.22\frac{\lambda}{D} \quad (3)$$

where $\lambda$ is the wavelength of the radiation you would like to observe, and $D$ is the diameter of the telescope. The factor of 1.22 is a dimensionless constant that comes from the use of Bessel functions in calculating the diffraction of the circular slit.