Optics Notes

Topics in these notes are

- wave properties
- Polarization
- focal lengths
- Geometrical optics

I. WAVE PROPERTIES

In general there are two types of waves. They are **transverse** and **longitudal** waves. Longitudal waves are those in which the displacement of the medium(or field) is in the direction of the wave. Examples of longitudal waves include sound waves and pressure waves. Transverse waves are waves in which the direction is perpendicular to the direction of motion. Examples of transverse waves include light and vibrations of a string.

The general equation for a wave is given by

$$y(x,t) = y_m \sin kx - \omega t \tag{1}$$

Where y_m is the maximum amplitude of oscillation, k is the wave number, and ω is the angular frequency. The relations of k and ω to physical quantities are given by.

$$k = \frac{2\pi}{\lambda} \tag{2}$$

$$\omega = 2\pi f \tag{3}$$

Where λ is the wavelength and f is the frequency given by.

$$f = \frac{\text{number of occurences of repeating event}}{time} = \frac{1}{T} \tag{4}$$

Where T is the period of the wave. Another usefull equation is:

$$\omega = \frac{2\pi}{T} \tag{5}$$

From these we can obtain the speed v of a wave. Namely,

$$v = \frac{\text{distance the wave travels in one period}}{period} = \frac{\lambda}{T} = \lambda f = \frac{\omega}{k} \tag{6}$$

The formulas for the speeds of different waves are given below.

1. Waves on a string

$$v = \sqrt{\frac{\tau}{\mu}}$$
$$\tau = \text{tension}$$
$$\mu = \text{mas per unit length}$$

2. E-M waves

$$v = \frac{c}{n}$$
$$c = \text{speed of light}$$
$$n = \text{index of refraction} \tag{7}$$

3. Sound waves

$$v = \sqrt{\frac{B}{\rho}}$$
$$B = \text{bulk modulus}$$
$$\rho = \text{density} \tag{8}$$

A. Reflections from a boundary

If a string wave is incident on a hard wall then the reflected wave is inverted. If the wave is free to move at the boundary then it is not inverted. The follwing url depicts the wave reflections well. `http://paws.kettering.edu/~drussell/Demos/reflect/reflect.html`

B. Standing waves

Standig waves do not propogate but just fluctuate in place. If a wave is confined to a cartain region of length L, then only certain frequencies are allowed to exist within it(see figure 1). The allowed frequencies depend on the boundary conditions. In general the two boundary conditions are **fixed** and **free**. For fixed boundary conditions we have $y(0,t) = y(L,t) = 0$. The free boundary conditions are $y'(0,t) = y'(L,t) = 0$, where $y(x,t)$ is the displacement at a time t and position x. When is comes to sound waves in a pipe **fixed end** = **closed end**, **free end** = **open end**.

By drawing allowed waves we can figure out the allowed wavelengths. For sound the allowed wavelengths for various conditions can be found by the following equations:

1. Both ends open

$$L = \frac{n\lambda}{2} \tag{9}$$

where $n = 1, 2, 3...$

2. One open end

$$L = \frac{n\lambda}{4} \tag{10}$$

where $n = 1, 3, 5...$

C. sound waves

The intensity I of a sound wave is given by;

$$I = \frac{1}{2}\rho v \omega^2 A^2 \tag{11}$$

Where ρ is the density of the medium, v is the velocity, ω is the angular frequency, and A is the amplitude of the wave.

The decibel level(or sound level) denoted by β is given by

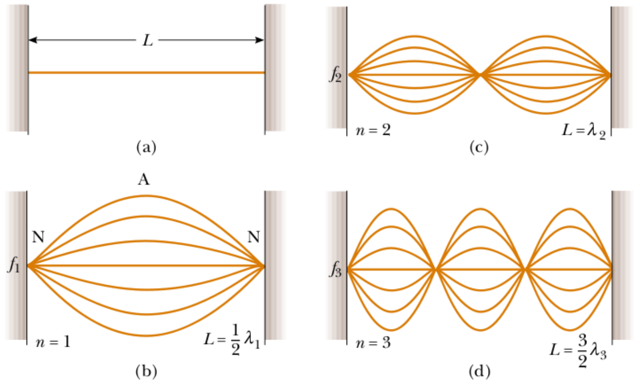

Figure 18.7 (a) A string of length L fixed at both ends. The normal modes of vibration form a harmonic series: (b) the fundamental, or first harmonic; (c) the second harmonic; (d) the third harmonic.

FIG. 1: depiction of a standing wave with both ends fixed

$$\beta = (10\text{dB}) \log \frac{I}{I_0} \tag{12}$$

Where dB is the decibel unit, I is the intensity of the sound, and $I_0 = 10^{12} W/m^2$.
For a **general** spherical wave the intensity falls off with distance as.

$$I(r) = \frac{P_s}{4\pi r^2} \tag{13}$$

Where P_s is the power generated by the source.

II. POLARIZATION

Polarization is defined as the direction in which the ELECTRIC field of an electromagnetic wave oscillates. For example if the electric field oscillates in the y-axis then the electromagnetic wave(light) is said to be y-polarized.

A linearly polarized E-M wave is one which the Electric and magnetic fields are confined to two orthogonal axes. With circular polarization the Electric and magnetic fields rotate with time.

A. Polarizers

A polarizer polarizes a light beam. Polarizing sheets(polarizers) have a **polarizing direction** or **polarization axis** which governs what polarizations the sheets will allow to pass through them. For example if a y-polarized beam is incident on a polarizing sheet that has it's polarization axis also in the y-direction then all of the light will be allowed through, it's intensity is unchanged. However, for an x-polarized beam no light will pass through(Intensity= $I = 0$). In general if the angle between the polarization of the light and the axis of the sheet is θ then the intensity of the beam after it has crossed the polarizer is given by (see Fig. 2)

$$I = I_0 \cos^2 \theta \tag{14}$$

Where I_0 is the initial intensity of the beam. NOTE: if the light wave that passes through the polarizer is unpolarized(randomly polarized) then the intensity of the light after it passes through the polarizer is reduced by 1/2.

$$I = \frac{I_0}{2} \tag{15}$$

For unpolarized light.

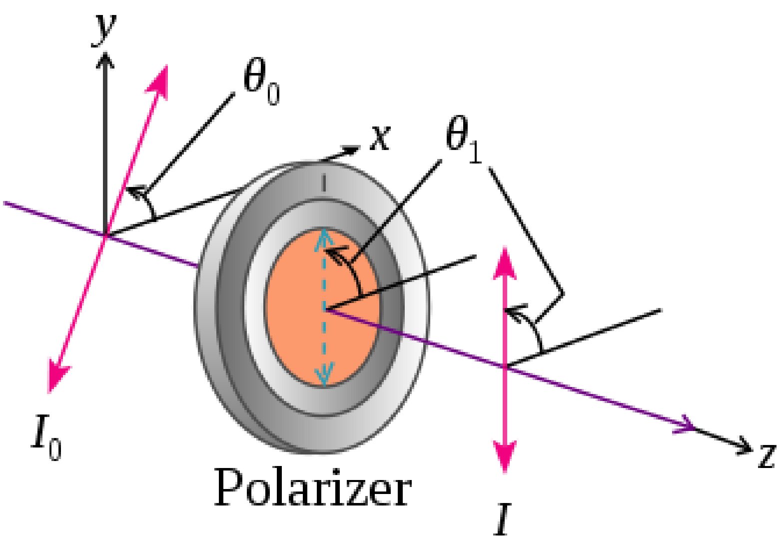

FIG. 2: Depiction of a standard polarizer problem. NOTE: $\theta = \theta_1 - \theta_0$

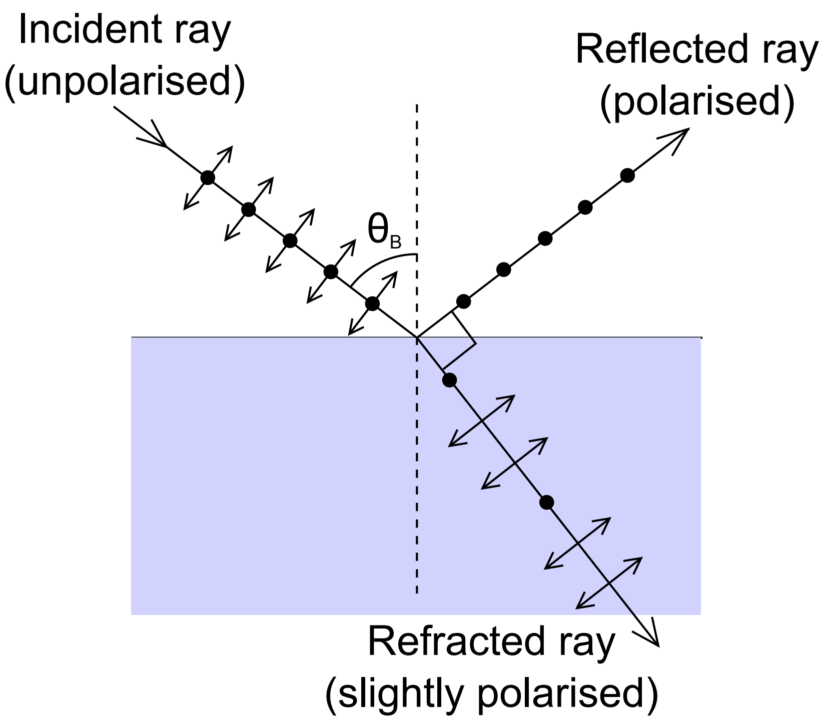

FIG. 3: Depiction of polarization using brewster angle

B. brewsters angle

Polarizers are not the only way one can polarize light. Another method is to have the beam incident on a surface at the brewster angle(see fig 3). If an unpolarized beam is incident at the brewster angle then the reflected beam is **plane polarized** (the electric field is out of the page). The brewster angle is given by

$$\tan\theta_B = \frac{n_2}{n_1} \tag{16}$$

Where n_2 is the bottom medium.

III. FOCAL LENGTHS

The importance of focal lengths is their use in geometrical optics which will be disussed in the next section. One will probably not need to calculate the focal length of a lens but if one does it will probably be a spherical mirror whose focal length is given by.

$$f = \frac{R}{2} \tag{17}$$

Where R is the sphere radius

IV. GEOMETRICAL OPTICS

A. Snell's Law

One of the fundamental equations in geometric optics is snell's law. It is given by

$$n_1 \sin\theta_1 = n_2 \sin\theta_2 \tag{18}$$

Where n_1 is the index of refraction of the 1st medium(where the incident light ray starts), n_2 is the index of refraction of the second medium, θ_1 is the angle of incidence, and θ_2 is the angle of refraction. NOTE: the angle of reflection is equal to the angle of incidence.

B. Plane Mirrors

For a plane mirror the object distance is equal to the image distance. NOTE: for a mirror the image distance is taken to be negative, namely;

$$q = -p \tag{19}$$

Furthermore if we define magnification as.

$$M = \frac{\text{Image height}}{\text{object height}} \tag{20}$$

For a plane mirror $M = 1$.

C. spherical mirrors

Definition: A virtual image is an image behind a mirror which usually has the same orientation as the object.
Definition: A real image is in front of a mirror and is upside down.
NOTE: The focal length for a concave mirror is positive and the focal length for a convex mirror is negative.
With these sign conventions the image distance can be obtained with

$$\frac{1}{p} + \frac{1}{i} = \frac{1}{f} \tag{21}$$

Where p is the object distance, i is the image distance, and f is the focal length. NOTE: the previous formula is the main formula in geometrical optics. Furthermore the magnification is given by,

$$M = \frac{-i}{p} \tag{22}$$

A plus sign in the magnification means the images have the same orientation.
We now lists the properties of these mirrors and their images.

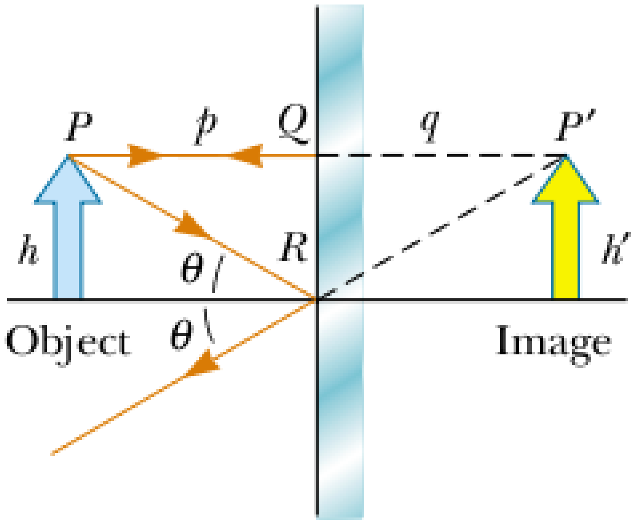

Figure 36.2 A geometric construction that is used to locate the image of an object placed in front of a flat mirror. Because the triangles PQR and $P'QR$ are congruent, $|p| = |q|$ and $h = h'$.

FIG. 4: Depiction of a plane mirror

Plane Mirror:
Image location = behind mirror
Image type = virtual
orientation = same as object
Convex Mirror:
Image location = behind mirror
Image type = virtual
orientation = same as object
sign of f = -1
Concave Mirror:
Image location = behind mirror (if $p < f$) in front of mirror (if $p > f$)
Image type = virtual (if $p < f$) real (if $p > f$)
orientation = same as object (if $p < f$) inverted (if $p > f$)
sign of f = +1

D. Ray Tracing for Spherical Mirrors

We can graphically locate the image of an object by drawing any two of the following four rays(NOTE: some rays are more advantageous to draw than others)

1. A ray that is initially parallel to the central axis will cross the focal point(the central axis is the axiswhich cuts the center of the spherical mirror)

2. A ray that passes through the focal point will reflect parallel to the central axis
3. A ray that passes through the center of curvature will reclect back on itself.
4. A ray that reflects off the intersection of the mirror with the central axis will reflect symmetrically(Not a very good ray)

The intersection of any of the two lines will give you the location of the image.

V. THIN LENSES

There are two types of lenses, converging, and diverging. These are also called convex and concave respectively. Just as before $i > 0$ if the image is real and $i < 0$ if the image is virtual. However, for lenses we must pay carefull attention to the meaning of real and virtual image. In the case of lenses the meaning is the reverse of that of mirrors.

DEFINITION: For a lens an image is virtual if it is formed on the side where the object is.

DEFINITION: For a lens an image is real if it is formed on the opposite side of the lens.

With these sign conventions the same equations that we had for a mirror hold for a lens.

We now list properties of the images formed by the two lenses

Convex(converging)

Image location = behind lens (if $p > f$) in front of lens (if $p < f$)
Image type = real (if $p > f$) virual (if $p < f$)
orientation = inverted (if $p > f$) same (if $p < f$)
sign of f = +1

Concave(diverging)

Image location = in front of lens
Image type = virtual
orientation = same as object
sign of f = -1

A. Ray Tracing

The three rays one can draw to find the object(remember one needs only two) are:

1. A ray that is initially parallel to the central axis of the lens will pass through the focal point behind the lens.
2. A ray that passes through the focal point in fron of the lens wil emerge from the lens parallel to the central axis.
3. A ray that is initially derected toward the center of the lens wil emerge from the lens with no change in its direction.

NOTE: For ray tracing there is no substitute for seeing lots of ray tracing diagrams.

B. Two lens systems

When looking for an image of an object that is in front of two lenses we must simply follow two steps. The steps are as follows

1. Ignore the second(farther) lens and find its image using Eq. (21)
2. Now ingoring the presence of lens 1 we treat the image found in step one as the object. NOTE: if the image found is one goes beyond the second lens then the object distance, p_2 is negative.

The magnification for a two lens system is

$$M = m_1 m_2 \tag{23}$$

Where $m_{1,2}$ are the magnifications produced by lens 1,2.

C. Sign Conventions

Because sign conventions are the trickiest part of geometrical optics I summarize them here.

$(f > 0)$

Concave mirror

Converging lens (convex lens)
($f < 0$)
Convex mirror
Diverging Lens (concave lens)
($i > 0$) (REAL IMAGE, Inverted)
Image in front of mirror
Image behind lens
($i < 0$) (VIRTUAL IMAGE, Same orientation)
Image behind mirror
Image in front of lens