GRE Mechanics Review

Society of Physics Students

October 18, 2004

1 Newtonian Mechanics

1.1 General Principles

The ideas of of Newtonian Mechanics are probably very familiar to you at this point, so here we will only summarize central laws and definitions:

- Linear momentum $\vec{p}$ and angular momentum $\vec{L}$ are conserved in all collisions, where

$$\vec{p} = m\vec{v} \tag{1}$$

$$\vec{L} = I\vec{\omega} = \left[\int r^2 dm\right]\vec{\omega} \tag{2}$$

- A collision in which energy is conserved in an *elastic collision* while one in which energy is not conserved is an *ineslatic collision*.

- Force and Torque are defined by:

$$\vec{F} = \frac{d\vec{p}}{dt} \tag{3}$$

$$\vec{\tau} = \frac{d\vec{L}}{dt} = \vec{r} \times \vec{F} \tag{4}$$

- *Static Equilibrium* is defined by the sum of all forces and all torques on an object being equal to zero.

1.2 Gauss's Law of Gravitation

If a surface S surrounds a mass whose density as a function of position is ρ and whose gravitational field is $\vec{g}$, then these two quantities can be related by Gauss's Law of Gravitation:

$$\oint_S \vec{g} \cdot d\vec{a} = 4\pi G M_{encl} = 4\pi G \int_V \rho dV \tag{5}$$

This formula is completely analagous to Gauss's Law for electric fields, replacing the electric field $\vec{E}$ with gravitational field $\vec{g}$ and $4\pi k$ with $4\pi G$, where $k = 1/4\pi\epsilon_0$ is Coulomb's constant and G is Newton's constant.

2 Fluid Mechanics

2.1 Continuity Equation

A fluid flowing in a pipe is subject to the continuity equation:

$$\rho A v = constant \tag{6}$$

where ρ is the mass density of the fluid, A the area of the pipe, and v the velocity of the fliud at each point. The continuity equation can be thought of as conservation of mass, and a physical example of this is illustrated in Figure 1(a).

2.2 Bernoulli's Equation

Fluid flow at different heights can also be

$$p + \frac{1}{2}\rho v^2 + \rho g h = constant \tag{7}$$

where again ρ is the mass density, v the velocity, y is the height, and p is the pressure on the fluid. Bernoulli's equation is essentially a consequence of energy conservation. An illustration of this is given in Figure 1(b)

2.3 Drag and Viscous Forces

For a spherical object of radius a moving through a fluid with velocity $\vec{v}$, the drag force is given by:

$$\vec{F}_{drag} = \frac{12\pi a^2 \rho v^2 \hat{v}}{R_e} \tag{8}$$

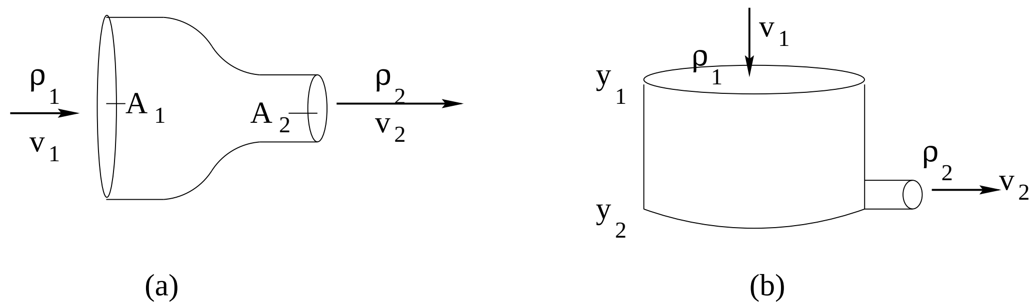

Figure 1: Graphic illustrations of (a) the continuity principle, and (b) Bernoulli's principle.

where ρ is the density of the fluid and R_e is the Reynolds number, which for a sphere is related to velocity by:

$$R_e = 2va/\nu \tag{9}$$

where ν is the *kinematic viscosity*, which is a characteristic of the fluid. This formula can be used to find the velocity of the particle so that one can calculate the drag force on a falling object if given the fluid's kinematic viscosity and density, as well as the object's radius and Reynolds number. In fact, in the sphere case discussed above, one can simplify the expression further to obtain:

$$\vec{F}_{drag} = 3\pi\rho R_e \nu^2 \tag{10}$$

3 Lagrangian and Hamiltonian Mechanics

3.1 Lagrangian Equations of Motion

In systems where using Newton's Second Law to determine the equations of motion is tedious or complex, one can use the formalism of Lagrangian mechanics as an alternate method to find the equations of motion. This method begins by determining Lagrange's function for the system:

$$L = T - U \tag{11}$$

where T is the kinetic energy of the system and U is the potential energy of the system. The Lagrangian, like T and U, is a function of position. As a particle moves along a path C, one can define the quantity I, called action, as:

$$I = \int_C L(x,t)dt \tag{12}$$

The central principle underlying Lagrangian mechanics is that a particle subject to a Lagrangian L will travel along the *path of least action* C_0 such that $I_0 = \int_{C_0} Ldt$ is the minimum value of I. If the system of coordinates is q_i then this condition leads to the equation:

$$\frac{\partial L}{\partial q_i} - \frac{d}{dt}\frac{\partial L}{\partial \dot{q}_i} = 0 \tag{13}$$

where $\dot{q}_i$ is the time derivative of coordinate q_i.

3.2 Hamiltonian Mechanics

Another formalism for solving mechanism problems is the Hamiltonian method. First one must form the familiar Hamiltonian:

$$H = T + U \tag{14}$$

so that the Hamiltonian has the familiar interpretation of the total energy of a system. The exact definition of the Hamiltonian is in fact a bit more compilcated (as those who have taken 8.09 will know), but this definition will suffice for the purposes of taking the Physics GRE.

The Hamiltonian formalism is useful in determining relationships between coordinates q_i and their associated momenta p_i. Though the definition of the momenta can at times be complex, the forms you are likely to encounter on the GRE will be ones familiar to you, such as $p_x = mv_x$ for linear motion and $p_\theta = I\omega$ for circular motion. Hamiltonian formalism is useful for determining force laws using Hamilton's equations of motion:

$$\dot{q}_i = \frac{\partial H}{\partial p_i} \tag{15}$$

$$\dot{p}_i = -\frac{\partial H}{\partial q_i} \tag{16}$$

where $\dot{q}_i$ and $\dot{p}_i$ are the time derivatives of the coordinates and momenta, respectively.