Physics GRE Review Fall 2004
Classical Mechanics Problems

Massachusetts Institute of Technology
Society of Physics Students
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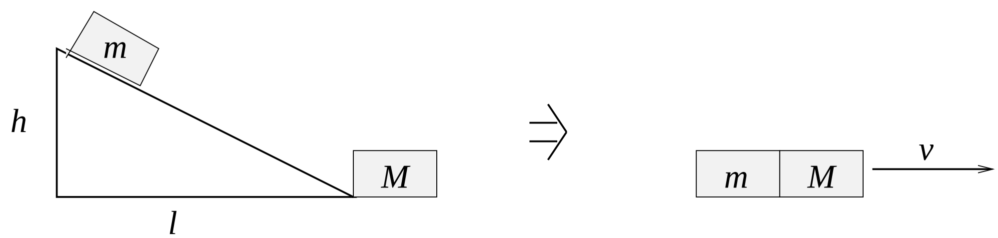
Classical Mechanics Problem Set

These problems are intended to help you review classical mechanics from 8.01 and 8.09, as well as material that may appear on the Physics GRE but is not covered in the standard MIT Physics curriculum.

1. **Energy and Momentum Conservation**
A mass m slides down a frictionless ramp of height h and length l. At the bottom, the mass collides and sticks to a larger mass M. Find the velocity v at which the two blocks travel after collision.

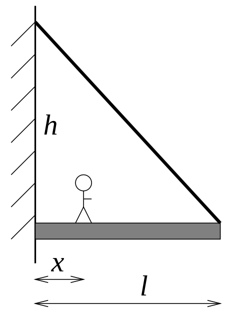


(a) $\sqrt{2gh}m/(m+M)$
(b) $\sqrt{2gl}m/(m+M)$
(c) $\sqrt{2gh}/\sqrt{h^2+l^2} * [m/(m+M)]$
(d) $\sqrt{2gh} * M/(m+M)$
(e) $\sqrt{2gh} * (m+M)/m$

2. **Ballistic Pendulum**
A bullet of mass m and velocity v_0 is shot into a wood block of mass M suspend on a string of length l. Find the maximum height the block will reach after collision.

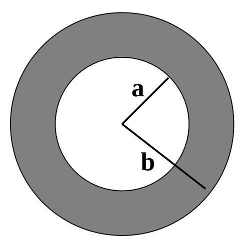
(a) $mv_0^2/[2g(m+M)]$
(b) $ml/(m+M)$
(c) $(mv_0)^2/(2gM^2)$
(d) $(mv_0)^2/[2g(m+M)^2]$
(e) $v_0^2/2g$

3. **Static Equilibrium**
   A beam of length l and mass m is supported by a wall-mounted pin at one end and at the other end by a cable attached to the wall a height h above the beam. A man of mass M stands a distance x from the wall on the beam. Find the tension in the cable.



   (a) $(ml/2 + Mx) * (h/\sqrt{h^2 + l^2})$
   (b) $(m/2 - Mx/l) * (l/\sqrt{h^2 + l^2})$
   (c) $(m/2 + Mx/l) * (h/\sqrt{h^2 + l^2})$
   (d) $(ml/2x + Mh/l) * (h/\sqrt{h^2 + l^2})$
   (e) $(m/2 - Mx/l) * (h/\sqrt{h^2 + l^2})$

4. **Gauss's Law of Gravitation**
   A spherical shell with inner radius a and outer radius b has a uniform mass density ρ. Find the gravitational field within the region $a \leq r \leq b$.



   (a) $(4\pi/3)G\rho r$
   (b) $(4\pi/3)G\rho(b^3 - a^3)/r^2$
   (c) $(4\pi/3)G\rho r^3/a^2$
   (d) $(4\pi/3)G\rho(r - a^3/r^2)$
   (e) $(4\pi/3)G\rho(r - b^3/r^2)$

5. **Stellar Gravitation**
   Two stars in a binary system orbit one another with a period T at a distance a. Find the mass M of each star.
   (a) $\pi^2 a^3/GT^2$
   (b) $2\pi^2 a^3/GT^2$
   (c) $2\pi^2 a^2/GT^3$
   (d) $4\pi^2 a^3/GT^2$
   (e) $4\pi^2 T^3/Ga^2$

6. **Fluid Dynamics**
   An incompressible fluid is stored in a cylindrical tank of radius $R_t = 5m$ and height $h_t = 3m$. Fluid is allowed to escape from a valve of radius $R_v = 5cm$ at the bottom of the tank. Find the rate at which the fluid level in the tank is dropping.
   (a) $5.2 * 10^{-4} m/s$
   (b) $6.3 * 10^{-4} m/s$
   (c) $7.7 * 10^{-4} m/s$
   (d) $8.5 * 10^{-4} m/s$
   (e) $9.2 * 10^{-4} m/s$

7. **Viscous Drag**
   A sphere of radius $a = 0.03cm$ with a Reynolds number of $R_e = 0.5$ falls through a fluid with kinematic viscosity $\nu = 0.12cm^2/s$ and density $\rho = 1.8 * 10^{-3} g/cm^3$. Find the drag force on the sphere.
   (a) $1.19 * 10^{-2} N$
   (b) $8.62 * 10^{-4} N$
   (c) $4.25 * 10^{-5} N$
   (d) $5.31 * 10^{-8} N$
   (e) $6.79 * 10^{-10} N$

8. **Lagrangian Mechanics**
   Find the Lagrangian that gives rise to the following force law: $F = m\ddot{x} = -\alpha(3x^3 + 2x)$ with $\alpha > 0$
   (a) $\frac{3\alpha}{4}x^4 + \alpha x^2$
   (b) $\frac{3\alpha}{4}x^4 - \alpha x^2$
   (c) $\frac{1}{2}m\dot{x}^2 + \frac{3\alpha}{4}x^4 + \alpha x^2$
   (d) $\frac{1}{2}m\dot{x}^2 - \frac{3\alpha}{4}x^4 - \alpha x^2$
   (e) $\frac{1}{2}m\dot{x}^2 - \frac{3\alpha}{4}x^4 + \alpha x^2$

9. **Hamiltonian Mechanics**
   Find the Hamiltonian that gives rise to the following force law: $F = dp_x/dt = -\beta(x^5 + 2x^3)$, with $\beta > 0$

   (a) $\frac{m\dot{x}^2}{2} + \beta(\frac{x^6}{6} + \frac{x^4}{2})$

   (b) $\frac{m\dot{x}^2}{2} - \beta(\frac{x^6}{6} + \frac{x^4}{2})$

   (c) $\frac{p_x^2}{2m} + \beta(\frac{x^6}{6} + \frac{x^4}{2})$

   (d) $\frac{p_x^2}{2m} - \beta(\frac{x^6}{6} + \frac{x^4}{2})$

   (e) $\frac{p_x^2}{2m} + \beta(\frac{x^6}{6} - \frac{x^4}{2})$