

1 Electromagnetism

General comments: the GRE is in SI units, so for those who took 8.022, make sure to pay attention to where the epsilons and mus go. (And thus the slightly different form of Maxwell's Equations.)

Also, on one of the practice tests I looked at, they asked about which if Maxwell's equations would need to be modified to account for magnetic monopoles.

Work: $\frac{W}{q_0}$ has units of volts. Volt = Joules/Coulomb.

1.1 Electricity

$$k = \frac{1}{4\pi\epsilon_0} \quad [\text{Nm}^2\text{C}^{-2}], \epsilon_0 \text{ is permittivity of free space}$$

$$F = k \frac{q_1 q_2}{d^2} \quad \text{Force between two charges}$$

$$F = q_1 E \quad \text{Force on charge in field}$$

$$E = 4\pi k \frac{Q}{A} \quad \text{Uniform electric field between two charged parallel plates, each of area } A$$

$$\vec{E} = k \frac{q\vec{r}}{r^2} \quad \vec{r} \text{ in direction } + \rightarrow -. \text{ Electric field of single charge.}$$

$$\frac{W}{q} = Ed \quad \text{Electric potential}$$

$$V = IR \quad \text{Ohm's}$$

$$P = VI = I^2R \quad \text{Power}$$

$$R = \rho \frac{l}{A} \quad \text{Resistivity}$$

$$R_{tot} = R_1 + R_2 + \dots \quad \text{Resistors in series}$$

$$\frac{1}{R_{tot}} = \frac{1}{R_1} + \frac{1}{R_2} + \dots \quad \text{Resistors in parallel}$$

1.2 Magnetism

$$\vec{F} = \vec{E}q + \vec{v}q \times \vec{B} \quad \text{Lorentz Force Law}$$

$$\vec{F} = I\vec{l} + \times \vec{B} \quad \text{Force on current carrying conductor}$$

$$B = \frac{\Phi}{A} \quad \text{Magnetic induction, } \Phi = \text{magnetic flux}$$

$$R = \frac{mv}{Bq} \quad \text{Radius of curvature of moving charge in magnetic field}$$

$$\vec{B} = \frac{\mu_0}{4\pi} \frac{q\vec{v} \times \hat{r}}{r^2} \quad \text{Magnetic field of moving charge}$$

$\oint B \cdot dl = \mu_0 I$ Ampere's law, use for calculating current

1.3 Capacitors

$C = \frac{Q}{V} = \epsilon_0 \frac{A}{d}$ Capacitance, $Q =$ charge on *either* plate. [farads]

$U_C = \frac{1}{2} CV^2$ Energy of Capacitor

$V = \frac{Q}{\epsilon_0 A} d$

Capacitors in series (or multiple different dielectrics): $C = \frac{1}{\left(\frac{1}{C_1}\right) + \left(\frac{1}{C_2}\right)}$

Charging: $q = (C\epsilon) (1 - e^{-t/RC})$

Discharging: $q = (C\epsilon) (e^{-t/RC})$

1.3.1 Undriven

RL Circuit: $V_R + V_L = RI + L \frac{dI}{dt} = 0$

$I(t) = I_0 e^{-Rt/L}$

RC Circuit: $V(t) = V_0 e^{-t/RC}$

1.3.2 Driven RLC Circuit

$V(t) = \underbrace{V_f}_{\text{forced response}} + \underbrace{Ae^{S_1 t} + Be^{S_2 t}}_{\text{natural response}}$

Where $S_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$

$\alpha = \frac{1}{2RC}, \omega_0 = \frac{1}{\sqrt{LC}}$.

1.4 Maxwell's Equations

Gauss's Law	$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0}$
Gauss's law for magnetism	$\vec{\nabla} \cdot \vec{B} = 0$
Faraday's law	$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$
Ampere's Law	$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J} + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}$

NOTE: \vec{J} is the total current density.

1.5 Magnetic and electric fields in matter

$\epsilon = \epsilon_0 \epsilon_r$ for media other than free space, where $\epsilon_r =$ relative permittivity of the media.
SIMILAR FOR MAGNETS?

1.6 Bio-Savart Law

$$\frac{dB}{dl} = \frac{\mu_0 I \sin \theta}{4\pi r^2}$$
$$\mathbf{B} = \int \frac{\mu_0 I d\mathbf{l} \times \hat{\mathbf{r}}}{4\pi r^2} = \int \frac{\mu_0 I d\mathbf{l} \times \mathbf{r}}{4\pi r^3}$$

$\hat{\mathbf{r}}$ is in direction from wire element to point where field being calculated.
PAY ATTENTION TO WHETHER OR NOT THIS IS RELEVANT?!
Bio-Savart is used for STEADY current.

1.7 Inductance

$$L = \mu_0 A \frac{N^2}{l} \quad \text{Inductance}$$

$$U_L = \frac{1}{2} LI^2 \quad \text{Energy of Inductor}$$

$$B = \mu_0 I \frac{N}{l}, \quad N = \text{number of coils, } l = \text{length of solenoid, [tesla], Ampere's law}$$

$$V_L = -L \frac{dI}{dt} \quad \text{Induced voltage Emf} = -\frac{d\Phi}{dt} \quad \text{Faraday's law, } BA = \Phi$$

$$E_{\text{induced in 2}} = -M \frac{dI}{dt} \quad \text{Mutual inductance. } M = \frac{N_2 \Phi_2}{i_1}$$

$$L \frac{dI}{dt} = N \frac{d\Phi}{dt} = -\text{Emf} \quad \text{Self inductance } L = \mu_0 \frac{N^2 A}{l} \quad \text{Solenoid AND Toroid}$$

1.8 Kirchoff's Laws

1st: Current into a junction = current out of a junction.

2nd: $\Sigma V = 0$ for any closed loop.